Demystifying Type II and Type III Compensators Using Op-Amp and OTA for DC/DC Converters

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ABSTRACT

The error amplifier for frequency compensation can be a simple voltage-to-voltage amplification device, that is, the traditional Op-Amp. This type of Op-Amp requires local feedback (between its output and inputs) to make it stable. And also, the error amplifier can be a voltage-to-current amplification device, that is, the gm Op-Amp (Operational Transconductance Amplifier, OTA). This is an open-loop amplifier stage with no local feedback. This application report describes how to select the placement of compensation poles and zeros properly using Op-Amp and OTA for both Type II and Type III compensators.

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1 Introduction

The purpose of adding compensation to the error amplifier is to counteract some of the gains and phases contained in the control-to-output transfer function that could jeopardize the stability of the power supply. Obviously, the ultimate goal is to make the overall closed-loop-transfer function (control-to-output cascaded with the error amplifier) satisfy the stability criteria. This is to avoid having the closed-loop phase any closer to 360 degrees than the desired phase margin anywhere the gain is greater than 1 (0 dB). It is also desirable to have the slope of the gain curve at the crossover point with a value of –20 dB/decade. Phase margins of 45 degrees to 60 degrees (360 degrees minus the total closed-loop phase lag) are considered safe values that yield well-damped transient load response.

Figure 1 shows the schematic of the traditional Op-Amp with Type I configuration. The operational amplifier (the traditional Op-Amp) represents the basis of the closed-loop system. Its function, in a feedback system, is to amplify the error detected between a fixed and stable reference level and the monitored state variable
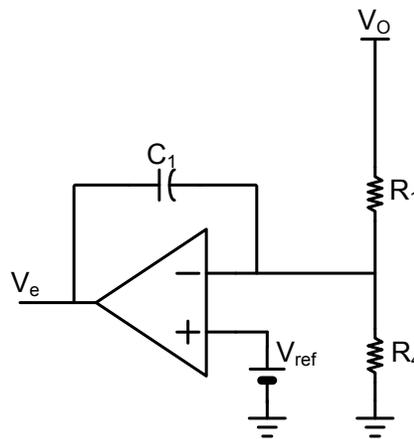
Figure 1.   Type I Compensator Using the Traditional Op-Amp

This type of Op-Amp requires local feedback (between its output and inputs) to make it stable. Under steady DC conditions, both the input terminals are virtually at the same voltage level. This determines the output voltage setting. However, though both resistors of the voltage divider affect the DC level of the converter's output, from the AC point of view, only the upper resistor enters into the AC analysis. So the lower resistor (R_4) is considered just a DC-biasing resistor, and therefore we usually ignore it in control loop (AC) analysis.

In this Type I configuration, we derive H(s) by dividing the capacitor impedance (C_1) by the upper resistor (R_1):

$$H(s) = \frac{V_e(s)}{V_o(s)} = -\frac{1}{sC_1R_1} \qquad (1)$$

A Type I circuit built on an OTA appears in Figure 2. The voltage on the inverting pin is the converter output voltage (V_o) undergoing the voltage division brought by R_1 and R_4, and the amplifier output voltage (V_e) is the output current multiplied by C_1 impedance.
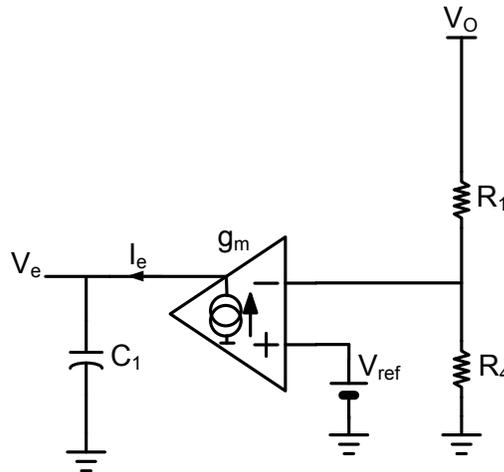
Figure 2. Type I Compensator with an OTA

The transfer function with an OTA in this Type I configuration is given by:

$$H(s) = \frac{V_e(s)}{V_o(s)} = -\frac{R_4}{R_1+R_4} \times g_m \times \frac{1}{sC_1} \qquad (2)$$

The biggest change compared to the conventional Op-Amp approach is the divider network and the OTA g_m parameter now enters into the equations. Regarding the divider network in the Op-Amp case; thanks to the virtual ground effect, both pins were at a similar potential and the ac contribution of the lower resistor (R_4) was nonexistent. In an OTA, there is no local feedback from V_e to the inverting pin, hence the absence of virtual ground. Therefore, we cannot ignore R_4 any longer and the OTA g_m parameter can vary depending on the care put in the integrated circuit design.

Note that the OTA model is really the simplest possible model, a voltage-controlled current source. A more complex model would feature upper and lower voltage clamp levels (given by the supply of the controller) plus a maximum output current limit. However, these extra components do not affect the AC response of the whole loop, and we can stay with the simplest representation.

If we are using an OTA, only the ratio of the feedback resistors is important. We could, for example, have a combination of 1 kΩ/5 kΩ or 10 kΩ/50 kΩ, and so on. They would all create the same gain (attenuation), and the gain-phase plot would not change. If we are using the conventional Op-Amp, the upper resistor affects the gain-phase plot. If we change that, we will get entirely different gain-phase results. Keeping the ratio unchanged does not keep the gain-phase plot unchanged in this case.

In adjustable regulators with the conventional Op-Amp, if we want to change the output voltage, it is best to change the lower feedback resistor, keeping the upper resistor unchanged. That way, the DC-biasing changes, but not the gain-phase (AC) characteristics of the feedback section.

2   Type II Compensator Using Op-Amp

Offering an origin pole, one zero, and one high-frequency pole, the Type II compensator provides a phase boost up to 90 degrees. Figure 3 shows the electrical configuration, and the transfer function is obtained by calculating the impedance offered by the network placed in the Op-Amp feedback path (Z_f) and dividing it by the upper resistor (R_1).

$$H(s) = \frac{Z_f}{Z_i} = \frac{\left.(\frac{1}{sC_1}+R_2)\frac{1}{sC_3}\middle/((\frac{1}{sC_1}+R_2)+\frac{1}{sC_3})\right.}{R_1} \tag{3}$$
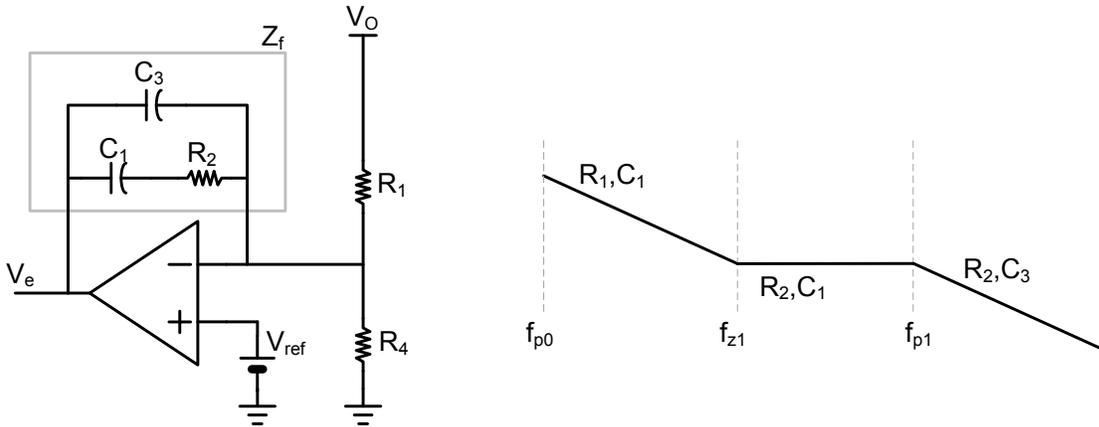
Figure 3.   Type II Compensator with Gain Curve of Op-Amp

Rearranging equation (3) leads us to the transfer function we are looking for:

$$H(s) = \frac{1+C_1 R_2 s}{(C_3+C_1)R_1 s + R_1 C_1 C_3 R_2 s^2} \tag{4}$$

Note that four components (R_1, R_2, C_1, and C_3) are involved in determining the poles and zero, and the locations of the poles and zero are:

$$f_{p0} = \tag{5}$$

$$f_{p1} = \frac{1}{2\pi \times R_2 \times C_3} \tag{6}$$

$$f_{z1} = \frac{1}{2\pi \times R_2 \times C_1} \tag{7}$$

We can find the required C_1, R_2, and C_3 once we select R_1 with the desired f_{p0}, f_{p1} and f_{z1}.

$$C_1 = \frac{1}{2\pi \times R_1 \times f_{p0}} \tag{8}$$

$$R_2 = \frac{f_{p0} \times R_1}{f_{z1}} \tag{9}$$

$$C_3 = \frac{f_{z1}}{2\pi \times R_1 \times f_{p0} \times f_{p1}} \tag{10}$$

Based on the locations of the poles and zero, Figure 4 shows an example for the compensator which has an appropriate shape, and usually a good phase margin.
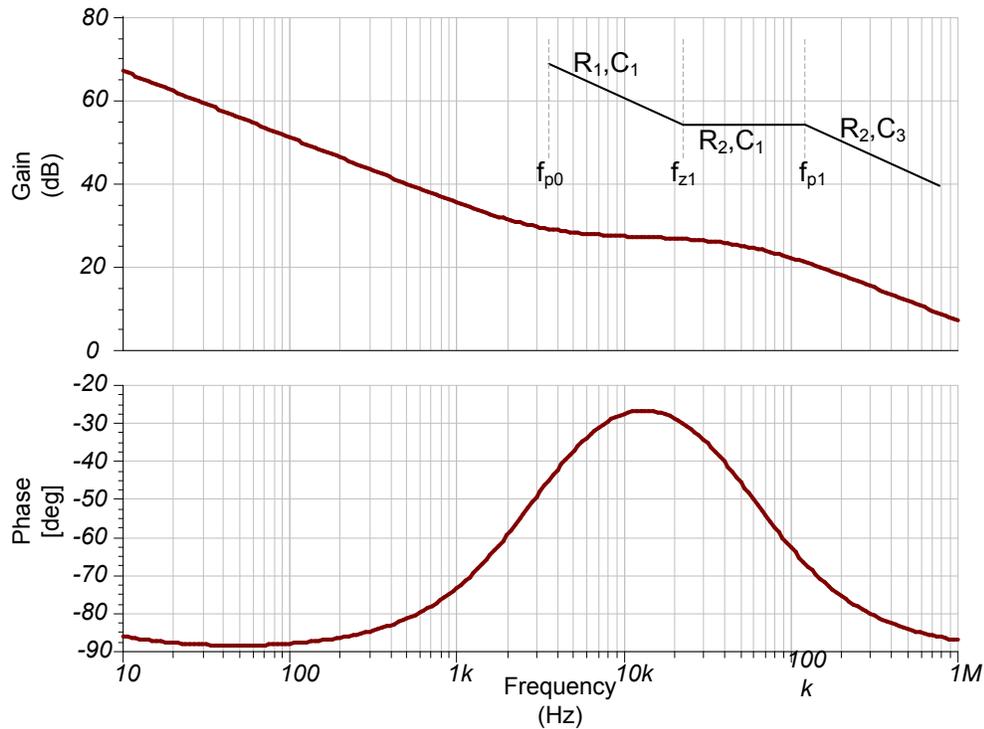
Figure 4.   Appropriate Shape of Type II Compensator

A Type II compensation amplifier adds an RC branch to flatten the gain, and improve the phase response in the mid-frequency range. The increased phase is achieved by increasing the separation of the pole and zero of the compensation.

Note that this type of compensator always has a net negative phase, and it cannot be used to improve the phase of the power stage. For this reason, Type II compensators cannot be used for voltage-mode control in CCM where there is a large phase drop just after the resonant frequency. Type II compensators are usually reserved for current-mode control compensation, or for converters that always operate in the DCM region.

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3   Type II Compensator Using OTA

We can visualize this feedback stage as a product of three cascade transfer functions, H1(s), H2(s), and H3(s) as shown in Figure 5. It combines a pole/zero pair plus an origin pole for a high DC gain, and the transfer function is defined as:

$$H(s) = \frac{V_e(s)}{V_o(s)} = H1(s) \times H2(s) \times H3(s) \tag{11}$$

So, the transfer function we are looking for:

$$H(s) = -\frac{R_4}{R_1+R_4} \times g_m \times \frac{1+R_2C_1s}{(C_3+C_1)s+R_2C_3C_1s^2} \tag{12}$$
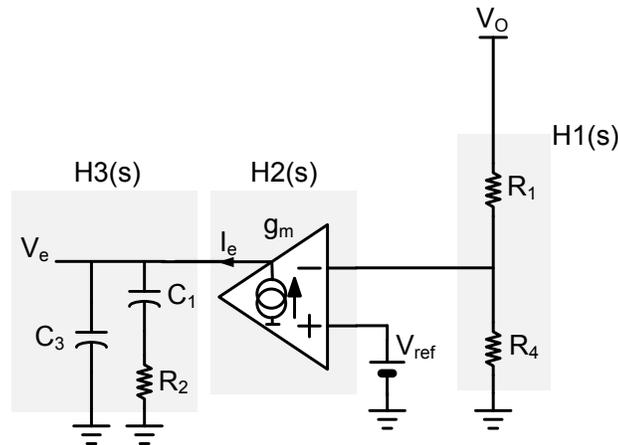
Figure 5.   Type II Compensator with OTA

If we factor sR_2C_1, we can easily unveil the poles and zeros definitions as follows:

$$H(s) = -\frac{R_4 g_m}{R_1+R_4} \times \frac{R_2 C_1}{C_1+C_3} \times \frac{1+\frac{1}{sR_2C_1}}{1+sR_2\frac{C_1C_3}{C_1+C_3}} = -G_0 \frac{1+\omega_z/s}{1+s/\omega_p} \tag{13}$$

In this equation, we can identify the three pertinent terms:

$$G_0 = \frac{R_4}{R_1+R_4}\frac{g_m R_2 C_1}{C_1+C_3} \tag{14}$$

$$f_z = \frac{1}{2\pi R_2 C_1} \tag{15}$$

$$f_p = \frac{C_1+C_3}{2\pi R_2 C_1 C_3} \tag{16}$$

To extract each individual value, we have to derive the magnitude of equation (13) at the crossover frequency (f_c):

$$|G(f_c)| = G_0 \frac{\sqrt{1+(\frac{f_z}{f_c})^2}}{\sqrt{1+(\frac{f_c}{f_p})^2}} \tag{17}$$

Combining (14), (15), (16), and (17), we find the following external components:

$$R_2 = \frac{f_p G (R_4 + R_1)}{(f_p - f_z) R_4 g_m} \times \frac{a}{b} \tag{18}$$

$$C_1 = \frac{1}{2\pi f_z R_2} \tag{19}$$

$$C_3 = \frac{R_4 g_m}{2\pi f_p G (R_4 + R_1)} \times \frac{b}{a} \tag{20}$$

Where:

$$a = \sqrt{1 + \left(\frac{f_c}{f_p}\right)^2} \tag{21}$$

$$b = \sqrt{1 + \left(\frac{f_z}{f_c}\right)^2} \tag{22}$$

G represents the gain or attenuation you are looking for at the selected crossover frequency (f_c). As a design example, if we need a 25 dB attenuation at 10 kHz with a phase boost of 50 degrees, selecting 100 µs transconductance of OTA (also, R1 = 40 kΩ, R4 = 25 kΩ), the external components (R_2, C_1 and C_3) are calculated as follows:

$$f_p = \left(tan\left(boost \times \frac{\pi}{180}\right) + \sqrt{tan\left(boost \times \frac{\pi}{180}\right)^2 + 1}\right) \times f_c = 27.5 \; kHz \tag{23}$$

As the phase peaks at the geometric mean between the pole and the zero, this letter is placed at:

$$f_z = \frac{f_c{}^2}{f_p} = 3.64 \; kHz \tag{24}$$

Applying the design definition from (18) through (22), we find

$$G = 10^{\frac{G_{fc}}{20}} = 10^{\frac{-25}{20}} = 0.056 \tag{25}$$

$$R_2 = \frac{27.5k \times 0.056(40k + 25k)}{(27.5k - 3.64k)25k \times 100\mu} \times \frac{\sqrt{(\frac{10k}{27.5k})^2 + 1}}{\sqrt{(\frac{3.64k}{10k})^2 + 1}} = 1.685 \; k\Omega \tag{26}$$

$$C_1 = \frac{1}{2 \times 3.14 \times 3.64k \times 1.685k} = 25.95 \; nF \tag{27}$$

$$C_3 = \frac{25k \times 100\mu}{2 \times 3.14 \times 27.5k \times 0.056(40k + 25k)} \times \frac{\sqrt{(\frac{3.64k}{10k})^2 + 1}}{\sqrt{(\frac{10k}{27.5k})^2 + 1}} = 3.96 \; nF \tag{28}$$

The results of this simulation with the given parameters appear in Figure 6 and show the –25 dB transition occurring at 10 kHz, as expected. Also, the boost in phase is 50 degrees as required.
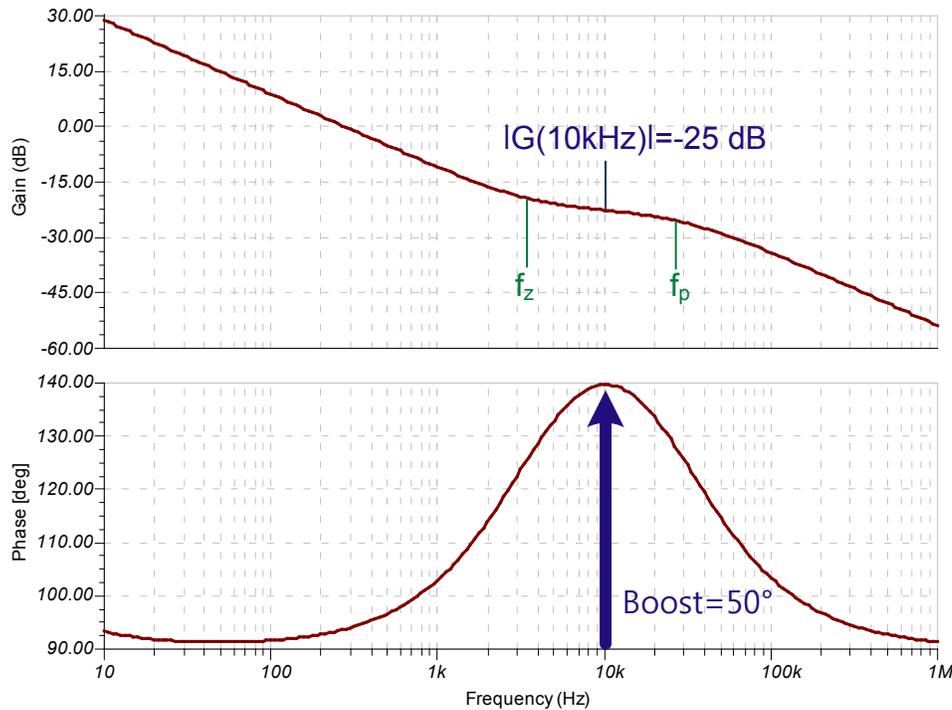
Figure 6. AC Simulation Plots of Type II Built with an OTA Confirm the Calculated Results

4 Type III Compensator Using Op-Amp

Figure 7 shows the conventional Type III compensation using voltage Op-Amp. There are two poles (f_{p1} and f_{p2}, besides the pole-at-zero f_{p0}) and two zeros (f_{z1} and f_{z2}) provided by this compensation. The Type III compensator is used when more than 90 degrees of phase boost are necessary. By adding another pole/zero pair to the Type II compensator, the Type III can theoretically boost the phase up to 180 degrees. The derivation of its transfer function does not really change, which means the principle remains the same with the Type II method.
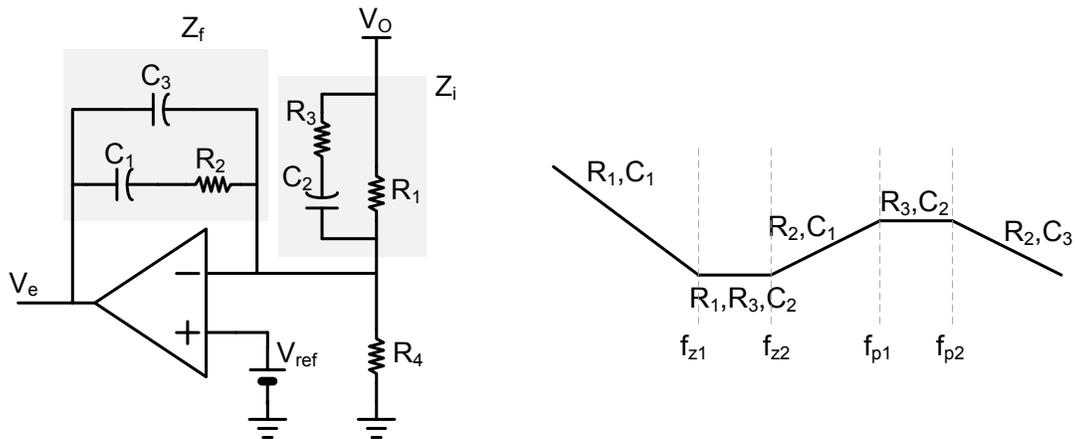
Figure 7. Type III Compensator with Gain Curve

We need to calculate the equivalent impedance Z_f placed across the Op-Amp, and divide it by that of Z_i to further rearrange the equation:

$$H(s) = \frac{Z_f}{Z_i} = \frac{\left.\left(\frac{1}{sC_1}+R_2\right)\frac{1}{sC_3}\middle/ \left(\left(\frac{1}{sC_1}+R_2\right)+\frac{1}{sC_3}\right)\right.}{\left.\left(\frac{1}{sC_2}+R_1\right)R_1\middle/\left(\left(\frac{1}{sC_2}+R_3\right)+R_1\right)\right.}$$

(29)

We obtain a more familiar expression:

$$H(s) = \frac{(sC_2(R_1+R_3)+1)(sC_1R_2+1)}{(sR_1(C_1+C_3))(sC_2R_3+1)(s\frac{C_1C_3R_2}{C_1+C_3}+1)} \approx \frac{(sC_2(R_1+R_3)+1)(sC_1R_2+1)}{(sR_1C_1)(sC_2R_3+1)(sR_2C_3+1)}, \quad if\ C_1 >> C_3$$

(30)

Note that several of the components involved play a dual role in determining the poles and zeros. So, the calculation can become fairly cumbersome and iterative. But a valid simplifying assumption that can be made is that C_1 is much greater than C_3. So the locations of the poles and zeros are finally:

$$f_{p0} = \frac{1}{2\pi \times R_1(C_1+C_3)} \approx \frac{1}{2\pi \times R_1 C_1}$$

(31)

$$f_{p1} = \frac{1}{2\pi \times R_3 C_2}$$

(32)

$$f_{p2} = \frac{1}{2\pi \times R_2 \left(C_1 C_3 / C_1+C_3\right)} = \frac{1}{2\pi \times R_2}\left(\frac{1}{C_1}+\frac{1}{C_3}\right) \approx \frac{1}{2\pi \times R_2 C_3}$$

(33)

$$f_{z1} = \frac{1}{2\pi \times (R_1+R_3)C_2}$$

(34)

$$f_{z2} = \frac{1}{2\pi \times R_2 C_1}$$

(35)

We can find the required C_1, C_2, C_3, R_2, and R_3 once we select R_1 with the desired f_{p0}, f_{p1}, f_{p2}, f_{z1}, and f_{z2} as follows.

$$C_1 = \frac{f_{p2}-f_{z2}}{2\pi \times R_1 f_{p0} f_{p2}}$$

(36)

$$C_2 = \frac{f_{p1}-f_{z1}}{2\pi \times R_1 f_{p1} f_{z1}}$$

(37)

$$C_3 = \frac{f_{z2}}{2\pi \times R_1 f_{p0} f_{p2}}$$

(38)

$$R_2 = \frac{R_1 f_{p0} f_{p2}}{(f_{p2}-f_{z2})f_{z2}}$$

(39)

$$R_3 = \frac{R_1 f_{z1}}{f_{p1}-f_{z1}}$$

(40)

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Based on the locations of the poles and zeros, Figure 8 shows a simulation example for the compensator which has an appropriate shape. This compensation scheme adds another RC branch to the Type II compensator. There is an integrator at low frequencies, followed by a pair of zeros. After this, the compensator has a region during where the gain increases with frequency, and the phase during this region is positive. In effect, the compensator is performing the function of a differentiator, measuring the slope of the power supply output waveform.
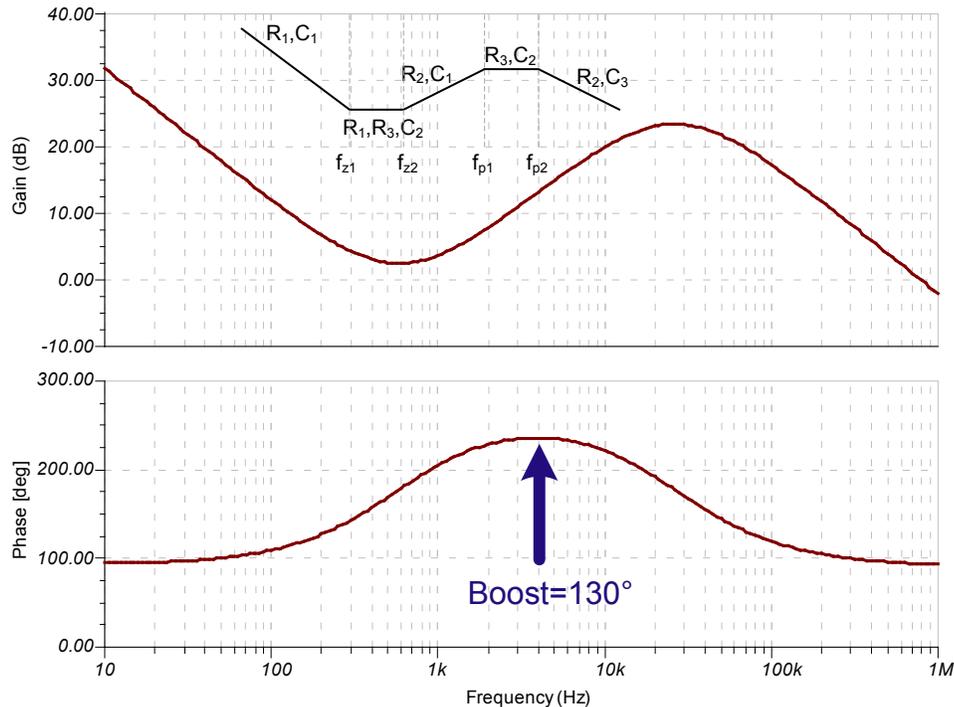
Figure 8.   AC Simulation Results of Type III Op-Amp Compensator

The phase boost in this case is about 130 degrees, but this can be increased by further separation of the poles and zeros. However, you cannot arbitrarily increase this pole-zero separation without considering the detrimental effects on shape of the loop gain. The Type III amplifier is the one for the voltage-mode converters operating in CCM.

5   Type III Compensator Using OTA

By combining an RC network across the upper resistor R_1, it is possible to build a Type III with an OTA. We can also visualize this feedback stage as a product of three cascade transfer functions, H1(s), H2(s), and H3(s) as shown in Figure 9. The transfer function is defined as:

$$H(s) = \frac{V_e(s)}{V_o(s)} = H1(s) \times H2(s) \times H3(s) \tag{41}$$

We know that the output current of the OTA depends on the voltage difference developed between its inputs. In AC, the voltage on the inverting pin depends on the divider network. So, the transfer function we are looking for:

$$H(s) = -\frac{R_4 + (R_1 + R_3)C_2 R_4 s}{R_1 + R_4 + (R_4 R_1 + R_3 R_1 + R_3 R_4)C_2 s} \times g_m \times \frac{1 + R_2 C_1 s}{(C_3 + C_1)s + R_2 C_3 C_1 s^2} \tag{42}$$
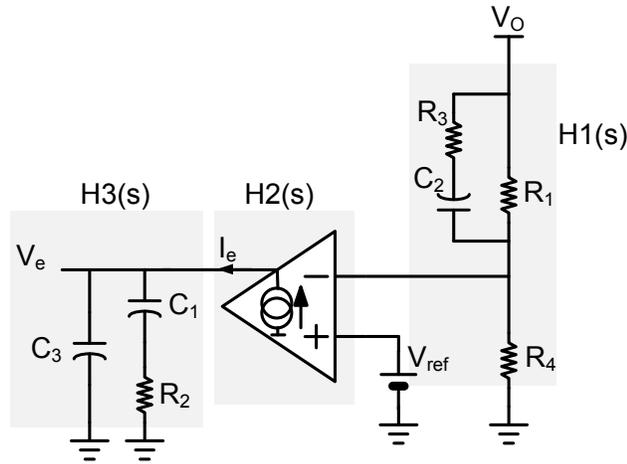
Figure 9.   Type III Compensator with OTA

Developing and arranging equation (42) leads to:

$$H(s) = -\frac{R_4 g_m}{R_4 + R_1} \times \frac{sC_2(R_3 + R_1) + 1}{(sC_2\left(\frac{R_4 R_1}{R_4 + R_1} + R_3\right) + 1)} \times \frac{1 + sR_2 C_1}{(s(C_1 + C_2)\left(1 + sR_2 \frac{C_1 C_3}{C_1 + C_3}\right))} \tag{43}$$

Factoring $1+sR_2C_1$, we have:

$$H(s) = -\frac{R_4 g_m R_2 C_1}{(R_4 + R_1)(C_1 + C_3)} \times \frac{1 + \frac{1}{sR_2 C_1}}{(sC_2\left(\frac{R_4 R_1}{R_4 + R_1} + R_3\right) + 1)} \times \frac{sC_2(R_3 + R_1) + 1}{(1 + sR_2 \frac{C_1 C_3}{C_1 + C_3})} \tag{44}$$

This expression can be put in the normalized form:

$$H(s) = -G_0 \times \frac{(1 + \frac{\omega_{z1}}{s})(1 + \frac{s}{\omega_{z2}})}{(1 + \frac{s}{\omega_{p1}})(1 + \frac{s}{\omega_{p2}})} \tag{45}$$

where:

$$G_0 = \frac{R_4 g_m R_2 C_1}{(R_4 + R_1)(C_1 + C_3)} \tag{46}$$

$$f_{z1} = \frac{1}{2\pi R_2 C_1} \tag{47}$$

$$f_{z2} = \frac{1}{2\pi(R_1+R_3)C_2} \tag{48}$$

$$f_{p1} = \frac{(C_1+C_3)}{2\pi R_2 C_1 C_3} \tag{49}$$

$$f_{p2} = \frac{1}{2\pi(\frac{R_4 R_1}{R_4+R_1}+R_3)C_2} \tag{50}$$

From the values, we need to extract the design expression that will let us calculate the passive values. We first need the magnitude of G at the selected crossover frequency:

$$|G(f_c)| = G_0 \times \frac{c\times d}{a\times b} \tag{51}$$

where:

$$a = \sqrt{(\frac{f_c}{f_{p2}})^2 + 1} \tag{52}$$

$$b = \sqrt{(\frac{f_c}{f_{p1}})^2 + 1} \tag{53}$$

$$c = \sqrt{(\frac{f_{z1}}{f_c})^2 + 1} \tag{54}$$

$$d = \sqrt{(\frac{f_c}{f_{z2}})^2 + 1} \tag{55}$$

$$aa = \frac{a\times b}{c\times d} \tag{56}$$

$$bb = \frac{G f_{p2}(R_1+R_4)}{R_4 g_m(f_{p2}-f_{z1})} \tag{57}$$

$$cc = |R_1^2 f_{z2} - R_1 R_4(f_{p1} - f_{z2})| \tag{58}$$

$$dd = (f_{p1} - f_{z2})(R_1 + R_4) \tag{59}$$

By using equations from (47) to (59), we can solve for R_2, R_3, C_1, C_2, and C_3:

$$R_2 = aa \times bb \tag{60}$$

$$R_3 = \frac{cc}{dd} \tag{61}$$

$$C_1 = \frac{1}{2\pi f_{z1} R_2} \tag{62}$$

$$C_2 = \frac{1}{2\pi f_{z2}(R_3+R_1)} \tag{63}$$

$$C_3 = \frac{C_1}{2\pi C_1 R_2 f_{p2}-1} \tag{64}$$

As a design example with a Type III compensator using OTA, if we want to stabilize a 12-V power supply obtained from 2.5-V reference voltage, the ratio between the output voltage and the reference is 12/2.5 = 4.8. To obtain 12-V output, we have R1 equal to 38 kΩ and R4 is 10 kΩ, and the pole (f_{p1}) and zero (f_{z2}) distance cannot be larger than 4.8.

When selecting the crossover frequency, you will have to make sure the maximum phase boost at the selected crossover point is possible (normally it is less than 150 degrees). Let's assume we want 130 degrees at a 1 kHz crossover frequency, a point where the loop needs 15 dB of amplification.

We can get the first zero and the second pole positions in relationship to the crossover frequency f_c:

$$f_{z1} = \frac{f_c}{\sqrt{130}} = 87.7\ Hz \tag{65}$$

$$f_{p2} = f_c \times \sqrt{130} = 11.4\ kHz \tag{66}$$

We know that the distance between the second zero and the first pole has to be less than 4.8 (4.5 is used for safety margin). Positioned in relation to the 1 kHz crossover frequency, they have to be placed at the following positions:

$$f_{z2} = \frac{f_c}{\sqrt{4.5}} = 456\ Hz \tag{67}$$

$$f_{p1} = f_c \times \sqrt{4.5} = 2.1\ kHz \tag{68}$$

Applying equations from (47) to (59), we find the following values for the passive elements placed around the OTA:

C_1 = 14.7 nF, C_2 = 9.2 nF, C_3 = 113.5 pF, R_2 = 123.9 kΩ, R_3 = 50 Ω

Once the AC sweep has been performed, the results appear in Figure 10.
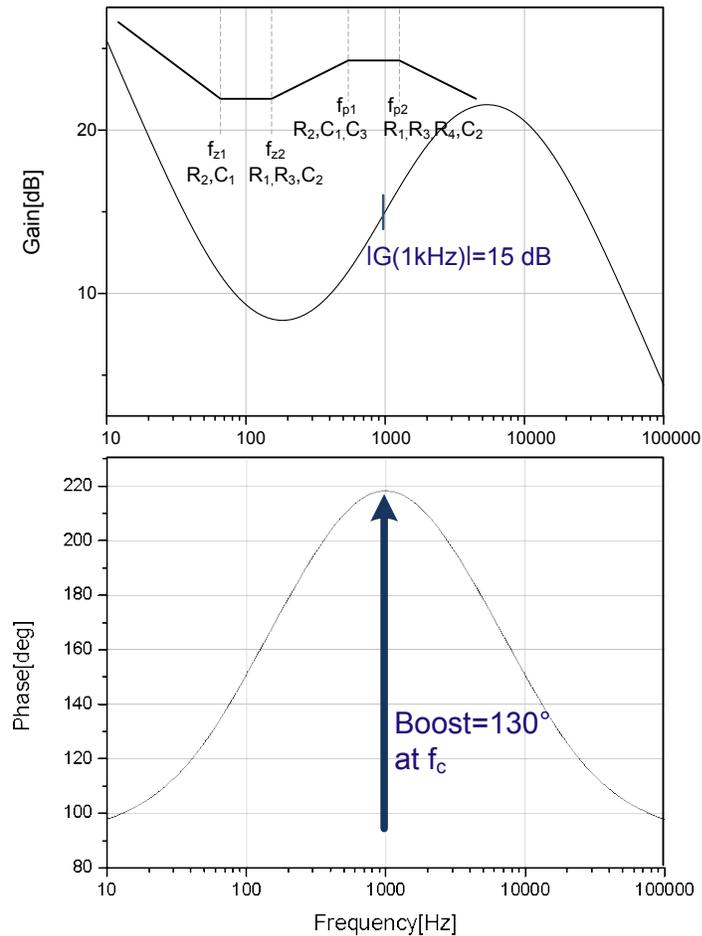
Figure 10. AC Response of Type III OTA

An OTA does not offer the same design flexibility as an Op-Amp. However, IC designers like the structure because of the small semiconductor die size it requires. The lack of virtual ground makes the lower side divider resistor enters the equations and requires attention when deriving poles and zeros placement. Depending on the ratio between the output voltage and the reference voltage, it can hamper the second zero (f_{z2}) and the first pole (f_{p1}) placement in a Type III configuration. This means that if we fix the first pole at some frequency, the location of the second zero is automatically defined. They are not independent. Therefore, there is no great flexibility in using this pole and zero. The phase boost of this type of compensator is needed to offset the sharp phase drop that occurs after the resonant frequency of the power stage.

6   Conclusion

Op-Amp requires local feedback (between its output and inputs) to make it stable and can be wired in a lot of different ways for control loop design. The lower resistor (R_4) does not affect the gain-phase plot in Op-Amp applications while the upper resistor (R_1) changes the gain-phase results. In adjustable regulators with Op-Amp, it is best to change the lower feedback resistor keeping the upper resistor unchanged if we want to change the output voltage. And also, the error amplifier can be a voltage-to-current amplification device, that is, the gm Op-Amp (Operational Transconductance Amplifier, OTA). This is an open-loop amplifier stage without local feedback. For OTA, both R_1 and R_4 enter into AC analysis, and note that only the ratio of the feedback resistors is important. Type II compensation scheme adds an RC branch to flatten the gain and improve the phase response in the mid-frequency range. Type II compensators are usually reserved for current-mode control compensation because it cannot be used to improve the phase of the power stage. Type III compensation scheme adds another RC branch to the Type II compensator and is the one to compensate voltage-mode converters operating in CCM. Note that a Type III compensator with an OTA does not offer the same design flexibility compared to the compensator with an Op-Amp because the ratio between the output voltage and the reference voltage can hamper the second zero and the first pole placement.

7   References

1. *Practical Feedback Loop Analysis for Voltage-Mode Boost Converter* (SLVA633),
    SW Lee, Texas Instruments, January 2014.

2. *Practical Feedback Loop Analysis for Current-Mode Boost Converter* (SLVA636),
    SW Lee, Texas Instruments, March 2014.

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